Variability of LiDAR volume prediction models for productivity assessment of radiata pine plantations in South Australia

J. Rombouts[1], I.S. Ferguson[2] & J.W. Leech[3]

[1]School of Forest and Ecosystem Science, University of Melbourne & Resource Planning Section, ForestrySA, rombouts.jan@forestrysa.sa.gov.au
[2] School of Forest and Ecosystem Science, University of Melbourne, iansf@unimelb.edu.au
[3]Forestry Systems, jleech@ozemail.com.au

Abstract

Site Quality (site productivity) information underpins many aspects of radiata pine plantation management in South Australia. The criterion of Site Quality is volume production at age 10 and is directly assessed by means of plot based and ocular assessments. Trials examining the use of LiDAR for Site Quality assessment were commenced in 2002. LiDAR data was captured using three different LiDAR systems in 2002, 2006, 2007 and 169 field plots were measured across 9 sites. A study was carried out to investigate the effect of LiDAR data capture parameters Campaign and Site on the regression relationships between forest and LiDAR variables. The study found that the factor Campaign had a significant effect on volume prediction models while a possible Site effect was detected for one Site. Predominant height prediction models were unaffected. Introducing Campaign and Site parameters in volume prediction models reduced Root Mean Square Error by up to 25.5%. Predominant height and volume prediction models explained 95.3% and 95.2% of the variance respectively. Campaign effects were not due to scanning angle, flying altitude or point density effects but appear to reflect differences in LiDAR systems and drought effects. Calibration protocols and modelling strategies are therefore needed for general application.

Key words: site productivity, LiDAR, modelling, stand volume, radiata pine

1. Introduction

Site Quality (site productivity) information underpins many aspects of radiata pine plantation management in South Australia. The productivity criterion used is total volume production to small end diameter underbark 10 cm, at or near age 10 y (Lewis *et al*., 1976). The Site Quality assessment method, in use since 1949, relies on objective (plot based) and subjective (ocular) assessments before any commercial thinning takes place. It results in a map showing seven Site Quality classes at a resolution of 0.1 ha (Figure 5).

The literature describes many examples of LiDAR estimation of stand volume (Maclean and Krabill, 1986; Nelson *et al*., 1988 and many others). Recognising that Site Quality assessment is a problem of assessing spatial variation in stand volume, studies were commenced in 2002 to test the feasibility of LiDAR based Site Quality assessment. The methodological framework adopted was the area based or height distribution method (Næsset, 2002). At the core of this method is the development of regression relationships between forest and LiDAR variables at the plot level.

Field data collection for calibration of prediction models constitutes a necessary and costly step in the method. The site and forest-type dependency of forest-LiDAR relationships has been the subject of several studies (Næsset *et al*., 2005; Lefsky *et al*., 2005; and others). These studies found that many forest–LiDAR relationships held across a broad range of sites and forest types when identical LiDAR instruments and comparable data collection parameters were applied. Changes in LiDAR systems and flight parameters may change the relationships between forest and LiDAR variables (Holmgren *et al.*, 2003, Lovell *et al*., 2005; Chasmer *et al*., 2006 and others).

The data for this study were collected in 2002, 2006 and 2007. Each trial contributed new sites and soil types. Each trial made use of a different LiDAR system. The objective of this study was to detect, describe and incorporate any "Site" and "Campaign" effects in the regression relationships between forest and LiDAR variables. The structurally homogenous, even aged, plantations of radiata pine comprising the study sites were particularly suited to the pursuit of this objective. As used in this text "Site" refers to the complex of soil, genetics, climate, silviculture and "Campaign" to the complex of LiDAR system and data capture parameters including seasonal effects.

2. Data and materials

2.1 Study sites

Figure 1 shows the location and rainfall at the nine study sites in the South East of South Australia. Because preferred assessment age would be between 8 and 10 plantations were selected in age range 7-11 (see Table 1). Sites were also selected so as to represent the main soil groups. In total 1756.2 ha of plantations were included in the study. All plantations were unthinned at time of data capture except for 100 metre wide Fuel Management Zones (FMZ) at the edges of some compartments in sites SP, MH, HO and DR.

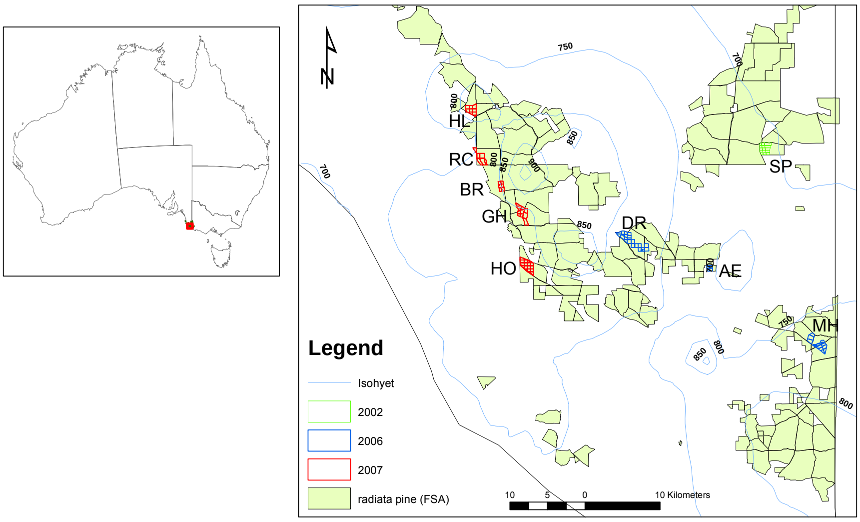

Figure 1: Study sites established in 2002, 2006 and 2007 in the South East of South Australia

2.2 Calibration plots

A total of 169 rectangular (20x25m) calibration plots were measured in 2002, 2006 and 2007. Plot locations were purposively selected so as to sample the full range of the Site Qualities and soil groups found on site. The grouping of soils followed Leech (1978) who identified 7 soil groups, each producing different growth patterns (Table 1 list soil groups in order of importance). At Site DR seven plots were located in the thinned FMZ. Plot corners were located by surveying along compartment boundaries and tree rows using measurement tape. Plantation boundaries had been surveyed by theodolite or differential GPS, with sub metre precision.

Table 1: Site specifics and measurement dates

Site	SP	DR	AE	MH	BR	RC	HL	HO	GH
Area (ha)	210.9	377.4	51.5	239.0	83.6	170.6	151.4	250.5	221.3
Soil Groups(*)	C, B	C, E, B	C, E	B	D, E	E, C	C, E	C, D, E	C, E
Measured	04/2002	04/2006	05/2006	05/2006	08/2007	07/2007	08/2007	05/2007	07/2007
Plantation age	10	9, 10	10	7, 9, 11	10	10	10	9,10	9

* B: Caroline sand; C: Other sand; D: Tantanoola flinty sand; E: Loamy sand and Terra Rossa

Table 2 shows that the number of plots measured per Site was poorly balanced with no replication of Sites across Campaigns.

Table 2: Number of plots measured by Campaign and Site

Campaign	Site									
	SP	DR	AE	MH	BR	RC	HL	HO	GH	Total
2002	28	0	0	0	0	0	0	0	0	28
2006	0	47	7	25	0	0	0	0	0	79
2007	0	0	0	0	9	13	8	17	15	62

Diameter at breast height and predominant height (PDH) were measured in the plots. In South Australia PDH is defined as average height of the 75 tallest trees per ha, restricted so that trees are evenly spaced in each quadrant of the plot. PDH was estimated as the average height of the 4 trees with largest diameter in each plot quadrant, increased by a constant of 0.45m to convert from largest to tallest (based on unpublished analysis).

Stand volume was predicted using a model fitted to stand volume data collected from Permanent Sample Plots (PSP) across the State. Some 372 measurements in stands aged 4-20 years old were used to calibrate a form-factor model predicting under-bark volume to small-end diameter 10cm underbark (V_{10}) with basal area, stocking, PDH and age as predictor variables. Figure 2 shows the range of PDH and V_{10} by Site.

2.3 LiDAR data and pre-processing

A different LiDAR system was used in each of the campaigns, as summarised in Table 3.

Table 3: LiDAR system details and flight parameters of three campaigns

	2002	2006	2007
System	Optech ALTM 3025	Leica ALS 50	Optech ALTM 3100
Date	7th July 2002	9th April 2006	20th July 2007
Flying altitude (m)	1,100	1,040	1,100
Pulse repetition rate (Hz)	25,000	73,200	33,000
Max scanning angle (dgrs)	15.0	13.5	12.5
Pulse density (m^{-2})	0.5-2.1	1.2-9.5	2.3-3.2
Returns per pulse	First and last echo	Up to four echoes	Up to four echoes

Data were captured at comparable flying heights and scanning angles. Point densities varied due to pulse repetition rates, number of fly-overs by the aircraft, scanner properties and data processing by the supplier. During Campaign 2006 higher point densities of up to 9.5 m^{-2} were recorded in narrow bands at the edges of flight strips, due to overlapping of strips as well as scanning mirror deceleration effects. Several calibration plots were located in those bands.

LiDAR returns were classified as ground or non-ground points by the supplier. The ground points provided the basis for the construction of a Digital Terrain Model (DTM) using an ESRI ArcGIS implementation of Delaunay triangulation. The height of LiDAR points above ground level was calculated as the difference between a point's z value and the z value of its projection on the DTM.

3. Methods

Ordinary Least Squares regression (OLS) is commonly used to calibrate prediction models in LiDAR applications. One of the base assumptions in OLS is that prediction errors are independent and normally distributed. In the presence of grouping structures in the data set, each with slightly different relationships between response and predictor variables, this assumption may be violated leading to biased estimates of the significance of predictor variables. Grouped data sets are a common occurrence in forest mensuration (for example multiple measurements on a single tree, remeasures of a permanent plot). Several strategies have been proposed to address the problem: data culling to minimise data grouping (Vanclay, 1994), stratifying data and fitting separate models to different strata (Næsset, 2002), introducing new explanatory variables including the use of dummy variables (Næsset *et al*., 2005), 2-stage modelling coupled with Generalised Least Squares techniques (Ferguson and Leech, 1978) and Mixed Effect Modelling (Gregoire *et al*., 1995; Breidenbach *et al*., 2007). Mixed-effects models are primarily used to describe relationships between a response variable and some covariates in data that are grouped according to one or more classification factors. The term mixed effects refers to the distinction between fixed effects (effects associated with an entire population or with certain repeatable levels of an explanatory variable) and random effects (localised effects associated with individual experimental units drawn at random from a population and regarded as additional terms, to account for correlation among observations within the same group), (Pinheiro and Bates, 2000).

Fixed and random effect variables have different roles: fixed effect variables explain variation while random effect variables help organise unexplained variation (Robinson, 2008).

Models were developed in two stages. In the first stage single predictor variable models were fitted for PDH and V_{10}, using OLS. Model residuals were analysed by Campaign and Site to reveal any structures indicative of grouping of the data. In the second stage one, two and three-predictor variable models were fitted, with incorporation of Campaign and Site effects, either as dummy variables in OLS models or as fixed or random effects in linear mixed effect (LME) models.

A range of LiDAR predictor variables were considered, describing different aspects of the distribution of laser heights in the calibration plots. To minimise the impact of differences in LiDAR system capabilities only the first return (i.e. first recorded echo) data were used. *All* first return data were used regardless of classification (ground or vegetation points) or pulse type (single or multiple return pulses). Variables included:

- mean height (*mh*), mean quadratic height (*mqh*), standard deviation of heights (*sdh)*
- maximum height (*hmax*), average of the maximum height in each plot quadrant (*hmax4*)
- percentile heights of the ordered cumulative height distribution (*h0, h10,…,h90*)
- proportion of ground returns (*propg*), proportion of returns in height frequency distribution classes (*d0, d10,…, d90*).

Most of these variables have been proposed in other studies. Additional predictor variables considered in the models were age, LiDAR point density and scanning angle.

To identify the most effective predictor variables a combinatorial screening approach was adopted whereby models were fitted to all possible combinations of the predictor variables mentioned above. Logarithmic transformations or quadratic forms of the variables were considered. The criterion for selecting the preferred one, two and three-variable models was the Akaike Information Criterion (AIC) – following Gregoire *et al*. (1995) - with the added constraint that each explanatory variable had to be significant at $p<0.05$. Root Mean Square Error (RMSE) was used as a measure of the precision of model predictions.

In OLS regression the dummy variables were used to distinguish Campaign and Site subgroups in the data set. Hypotheses of difference in slope and/or intercept dummy variables were compared.

In LME models Campaign was introduced as a fixed effect because of the low number of levels of the variable and its specific nature. Site was introduced as a random effect for reasons explained later. Several possible assumptions regarding the random effects were tested: (1) variable intercept but constant slope, (2) variable slope but constant intercept or (3) variable intercept *and* slope. Models with the same fixed effects but different random effect assumptions were compared using likelihood-ratio tests.

Analysis was performed using R statistical software (R-Development-Core-team, 2007).

Volume prediction models were used to generate volume surfaces which were then converted to Site Quality maps using volume to Site Quality conversion tables. LiDAR Site Quality maps were compared with conventional Site Quality maps using an error matrix approach. Because of space constraints the results of this analysis could not be reported here. However an example of a LiDAR and conventional Site Quality map was presented for illustrative purposes.

4. Results

4.1 Models without Site and Campaign effects

Single variable OLS models for PDH and V_{10} were fitted to the whole data set and residuals were examined. The best predictor for PDH (lowest AIC) was *hmax4* (the mean of the highest returns in each of the four plot quadrants). This was interesting because *hmax4* was the variable that most closely matched the way PDH was measured in the field. There was no evidence of curvature in the relationship. The best single predictor variable for V_{10} was *mqh* (mean quadratic height of first returns, including ground returns). This is a somewhat similar result to that obtained by Nelson *et al.* (2007) in loblolly pine plantations in the south eastern United States. The PDH and V_{10} models explained 95.3% and 91.1% of the variance in the data. Figure 2 shows the fitted models.

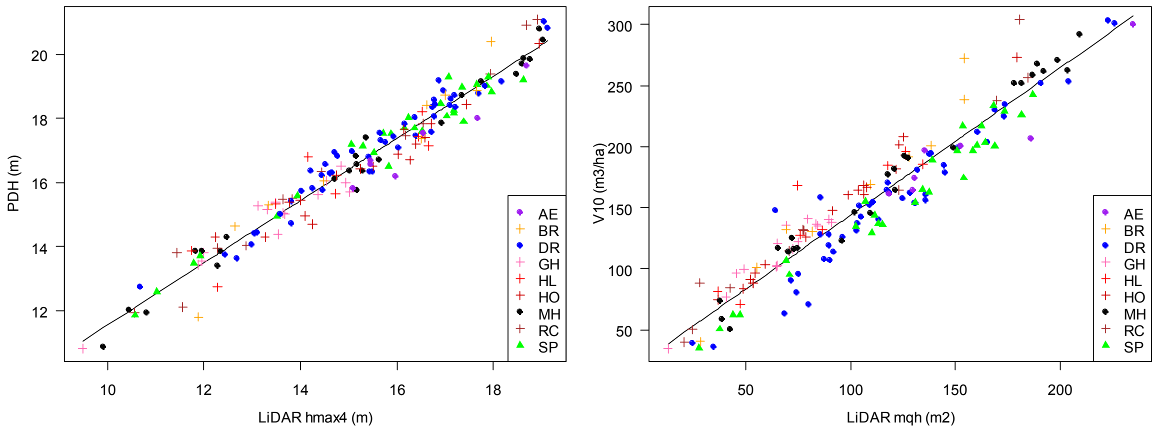

Figure 2: Single variable OLS models for PDH and V_{10}

The PDH model residuals were indifferent to the factor Campaign and evidence of Site effects was weak. The V_{10} model residuals however were strongly correlated with the factor Campaign. There were also significant differences at the Site level but those mostly mirrored Campaign trends, with the notable exception of Site MH (see Figure 3, left). Further analysis showed that the residuals of the models were not significantly correlated with scanning angle or LiDAR point density. Furthermore, plots located in the thinned FMZ of Site DR did not produce different residual patterns compared to unthinned plots and were therefore left in the data set (in fact this confirms the efficiency of the variable *mqh* as a predictor of volume).

It was concluded that Campaign, and possibly Site effects, were affecting volume-LiDAR relationships, suggesting that models in Figure 2 could be improved.

4.2 Models incorporating Site and Campaign effects

Comparison of AIC and likelihood ratio tests showed that Site and Campaign effects were not significant in PDH models and no further analysis was carried out of those models.

Dummy variables for Campaign and Site were introduced into a single variable OLS volume model. Table 4 shows 3 variants of this model. Models with Campaign dependent slope or Campaign dependent intercept *and* slope had a lower AIC indicating that a model structure consisting of parallel lines best fit the data. *C_2006, C_2007* and *S_MH* are dummy variables that take the value of 1 when Campaign is equal to 2006 or 2007, or Site is equal to “MH”. Otherwise they have a value of zero.

Table 4: Ordinary Least Squares volume prediction models incorporating Campaign and/or Site effects

Model	Parameters (standard error)	RMSE	AIC
1	V_{10} = 22.91 + 1.209 *mqh* (3.50) (0.029)	18.4	1472
2	V_{10} = -2.36 + 1.300 *mqh* + 8.49 *C_2006* + 30.77 *C_2007* (4.14) (0.025) (3.22) (3.46)	14.4	1388
3	V_{10} = 0.10 + 1.301 *mqh* + 28.25 *C_2007* + 15.89 *S_MH* (3.32) (0.024) (2.51) (3.18)	13.7	1372

Model 2 showed significant differences between all three Campaigns at p=95%. Model 3 expressed that without the data for Site "MH", the difference between Campaign 2002 and 2006 was no longer significant. While the variable *S_MH* indicated a possible Site effect, its significance had to be questioned given the limitations of the data set (such as no replication of Sites across Campaigns). There was no evidence of Site effects in Campaign 2007 despite the important soil differences between Sites (See Table 1). Analysis showed that the distribution of the Campaign 2007 model residuals was not correlated with Soil Groups.

Campaign and mqh were introduced into LME models as fixed effects. Site was introduced as a random effect because the OLS result for Site "MH" is difficult to rationalise against a range of anticipated effects from other Sites and the ill-balanced distribution of the data (see Table 2). Analysis of model fit parameters and log likelihood parameters showed that random effects were most effectively modelled as random intercepts (parallel lines). Table 5 shows three variants of the model. RMSE are provided both for global and localised (Best Linear Unbiased Predictors - BLUP) predictions.

Figure 3 shows the marked improvement of the distribution of residuals of models including Campaign and Site effects (OLS Model 3 and LME Model 2) compared to the model without these effects.

Table 5: Linear Mixed Effect volume prediction models incorporating Campaign and/or Site effects.

Model	Effects		RMSE		AIC
	fixed (standard error)	random	global	local	
1	V_{10} = 18.10 + 1.293 *mqh* (5.27) (0.024)	*Site*	19.3	13.7	1405
2	V_{10} = -2.26 + 1.299 *mqh* + 8.78 *C_2006* + 30.90 *C_2007* (6.20) (0.024) (6.37) (6.18)	*Site*	14.5	13.7	1382
3	V_{10} = 4.01 + 1.300 *mqh* + 24.59 *C_2007* (4.36) (0.024) (4.39)	*Site*	14.8	13.7	1388

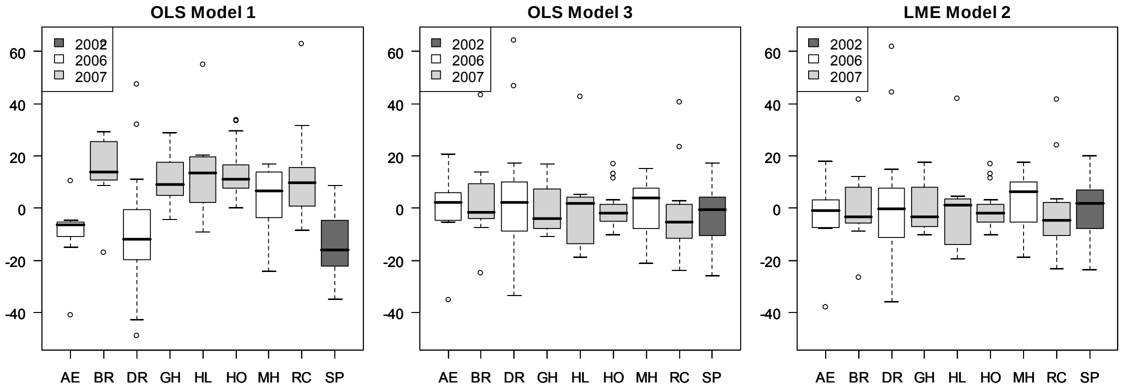

Figure 3: Residuals of OLS and LME volume prediction models grouped by Campaign and Site

Likelihood ratio tests comparing the LME models with their corresponding OLS form were all significant at the 95% probability level. Model 2 was the model with the lowest AIC, and interestingly the variable *C_2006* was not significant at 95%. According to this model therefore the difference between the Campaign 2002 and 2006 effects was not as important as indicated by corresponding OLS model 2. Given the data and the Campaign circumstances (rainfall in preceding year, see Discussion) the LME inference seems the more plausible one.

Incorporating Campaign and Site effects in prediction models reduced RMSE by up to 25.5%. Analysis not reported here showed that the inclusion of additional LiDAR variables failed to improve model fit and that Campaign and Site effects remained significant. OLS model 3 explained 95.2% of the variance, LME model 1 (BLUP) explained 95.1%.

Site Quality maps were compiled using LME, Model 1 and were compared with conventional Site Quality maps. An example is shown in Figure 4 and clearly demonstrates the potential of LiDAR as an alternative basis for Site Quality assessment.

5. Discussion

Significant Campaign effects were observed in the relationship between *mqh* and plot V_{10}. No such effects were observed in the relationship between *hmax4* and plot PDH. This indicated that Campaign effects affected the shape but not the range (minimum, maximum) of the distribution of LiDAR first returns heights observed in a plot. Differences in flying height, scanning angle and point density were rejected as possible explanations of these effects. There simply were no significant differences in flying height between Campaigns and there was no correlation whatsoever between scanning angle or point density and the model residuals of OLS, Model 1. The observations in this study were mostly consistent with the findings of Chasmer *et al*. (2006) who reported greater canopy penetration rates as laser pulse energy increased. However, that study considered *all* pulse echoes (up to four) rather than just the first echoes used in this study. A more plausible hypothesis perhaps was that the severe drought in the year leading up to Campaign 2007 reduced plantation leaf area and hence the relationship between *mqh* and plot V_{10}. Linder *et al*. (1987) found that in a period of drought radiata pine responds by producing shorter needles, as well as shedding significantly more older needles earlier in the summer. The rainfall in the 12 months preceding Campaign 2007 data capture was 554 mm while it was 762 and 707 mm in the year leading up to Campaign 2002 and 2006 respectively. Nelson *et al*. (2000) offered a similar explanation in a study in Costa Rica.

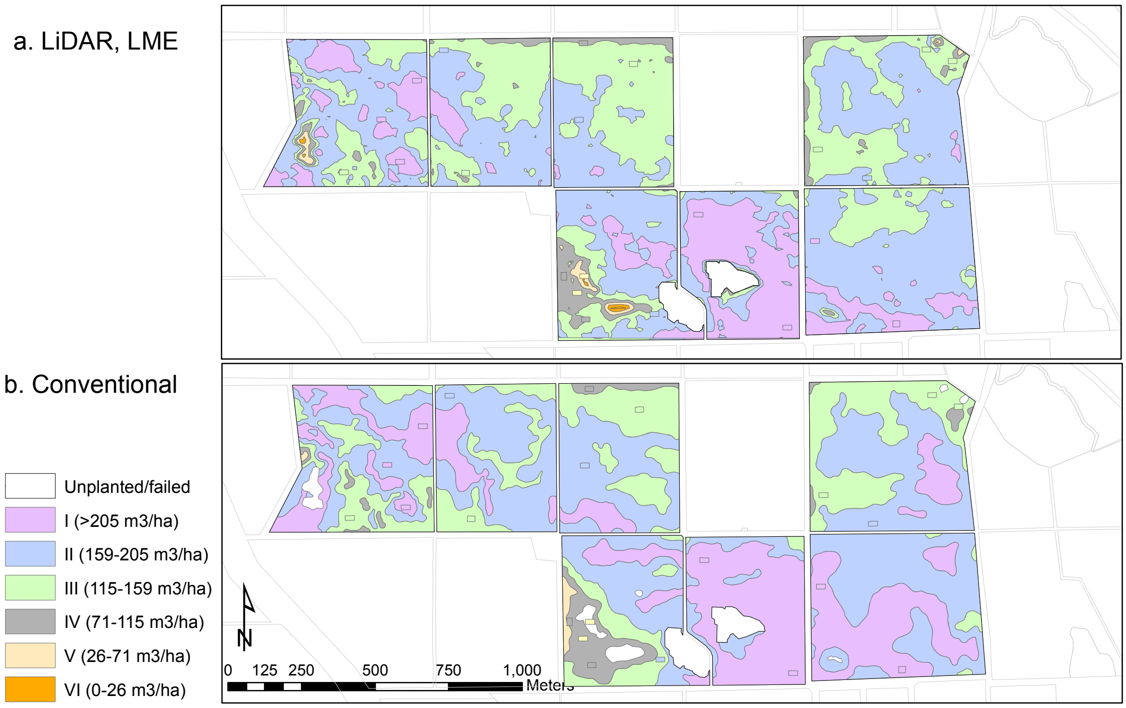

Figure 4: LiDAR and conventional Site Quality maps for Site DR, 9 year old plantation. Calibration plots are shown in a colour indicating field measured Site Quality.

The only evidence of a possible Site effect was observed in Site MH. It has been well documented that soil and climate differences may lead to site specific relationships between stand height and stand volume (Lewis *et al*., 1976; Skovsgaard and Vanclay, 2007). However, given the isolated occurrence of a Site effect in this study, given that there was no replication of comparable Site conditions in the study and given the ill-balanced distribution of data, treating Site as a random effect in a mixed effect model seemed a more prudent path for prediction.

Mixed effect modelling may also provide advantages where a model needs to be applied outside the model's domain (where Site is unknown) or when the number of dummy variables becomes unmanageable. The inclusion of LiDAR variables additional to *mqh* into the prediction models did not significantly improve the models. This is consistent with the findings of Nelson *et al.* (2007).

The objective of Site Quality assessment is to make site-specific, spatially explicit estimates of forest productivity. The Campaign and Site effects detected in this study can therefore not be ignored. Calibration data collection protocols need to be developed that produce the field data necessary to detect and quantify these effects. Alternative sampling strategies such as random, systematic and purposive sampling need to be compared to identify the strategy that best fits the purpose of the data, which is to fit a calibration model of known structure. Modelling techniques need to be adopted that allow for Campaign and Site effects to be expressed and make efficient use of calibration data collected in the past. Planned research aims to address these questions.

6. Conclusion

The study has produced evidence that Campaign and possibly Site effects influence the relationships between stand and LiDAR variables in young age radiate pine plantations. Of the two effects Campaign is by far the more important one; volume relationships were significantly affected while predominant height relationships were not. These effects were successfully incorporated in volume prediction models using Ordinary Least Squares and Mixed Effect modelling techniques resulting in reductions of RMSE by up to 25.5%. Hypotheses as to the cause of the effects were presented. The evidence indicates that Site and Campaign effects cannot be ignored in the calibration of LiDAR prediction models for Site Quality assessment and should be considered in field data collection protocols and modelling techniques. Research needs were highlighted.

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