Estimation of tree lists from airborne laser scanning data using a combination of analysis on single tree and raster cell level

Eva Lindberg[1], Johan Holmgren[1], Kenneth Olofsson[1],
Håkan Olsson[1] and Jörgen Wallerman[1]

[1] Swedish University of Agricultural Sciences, Department of Forest Resource Management, SE-901 83 Umeå, Sweden, (Eva.Lindberg)@srh.slu.se

Abstract

Airborne laser scanning produces high resolution data which opens up for estimation methods on individual tree level. However, the detection rate depends on the forest structure, and typically suppressed trees below a dominant tree layer are not detected. This paper presents a method to produce tree lists consistent with unbiased estimates on raster cell level. First, automatic delineation of tree crown segments was performed. The number and attributes of trees were estimated within segments. Second, forest variables were estimated on a field plot level using both laser canopy height distribution and results from tree detection. Percentiles of the stem diameter and tree height distributions were estimated using regression models. Third, the estimated percentiles were used as input for imputation of field trees from similar field plots in order to create a target distribution matrix. The number of trees in this matrix was estimated by scaling with the estimated total volume for each field plot. Finally, the initial tree list obtained from the tree crown segmentation was adjusted by using the estimated target distribution matrix. Random errors and bias for stem volume and stem number estimates could be reduced by combining analysis on tree and raster cell level.

Keywords: Lidar, single tree, area based, tree list, diameter distribution

1. Introduction

High resolution airborne laser scanning, ALS, data (≥10 measurements m^{-2}) can be used for analysis on a tree level (e.g. Hyyppä *et al.* 2001; Persson *et al.* 2002; Solberg *et al.* 2006). A digital height model is created from laser data and image analysis techniques, most often Individual Tree Crown delineation, ITC, are used to detect individual trees and measure position, height, and canopy shape. This method is now being marketed as operational. However, the detection rate depends on the forest structure (Persson *et al.* 2002). Thus, estimates that are based only on analysis of individual trees might be biased (Maltamo *et al.* 2004).

ALS is used operationally in Scandinavia for estimation of forest variables on raster cell level, the so called area based method, usually with regression models built on the Laser Canopy Height, LCH. The area based method generally produces forest variable estimates with high accuracy (Næsset *et al.* 2004) and low bias (Maltamo *et al.* 2006). Single tree methods usually have lower accuracy and underestimate the amount of trees (Næsset *et al.* 2004). Maltamo *et al.* (2004) have suggested a combination of methods to use the high accuracy from area based methods and the information for the dominant tree layer from single tree methods

The aim of this study is to develop methods to supply an information system with a list of trees, each tree with estimated attributes, e.g. stem diameter and tree height. A method is presented to estimate tree lists with a combination of individual tree and raster cell level estimates. The objective is to develop and validate a method that produces tree lists consistent with unbiased estimates on a raster cell level.

2. Material and Methods

2.1 Study area

The study area is 1989 hectare large and located in the north of Sweden (lat. 64^0 25' N, long. 14^0 50' E). The dominating tree species are Norway spruce (*Picea Abies*), birch (*Betula spp*) and Scots pine (*Pinus Silvestris*). The elevation ranges from 325 to 658 m a.s.l., which means that the site is located close to the limit for productive forest.

2.2 ALS data

The laser data acquisition was performed on August 7 and 8 2007 using a Leica ALS50-II airborne laser scanning system carried by a helicopter. The flying altitude was 600 m and the scan angle ±16 degrees, resulting in a scan width of 375 m and a scan density of about 10 points m^{-2}. Laser returns were classified as ground or non ground using a progressive Triangular Irregular Network (TIN) densification method (Axelsson 1999, 2000) in the TerraScan software (Soininen 2004), and the ground returns were used to derive a Digital Terrain Model (DTM).

2.3 Field data

The area was divided into five strata using an existing stand register and a total of 179 field plots were allocated (Table 1). The field plot radius was 6 m in stratum 1-3 and 8 m in stratum 4-5. The position of the field plots were measured using a Global Navigation Satellite System (GNSS). The trees on the field plots were measured using the Forest Management Planning Package (Jonsson *et al*. 1993). Within the plots, all trees with a stem diameter larger than the minimum stem diameter, 40 mm in stratum 1-3 and 60 mm in stratum 4-5, were callipered and tree species was recorded. The positions of the trees were registered relative to the centre of each plot by measuring azimuth and distance with a compass and ultrasonic device, respectively. The position of a tree was not measured if the tree had a large inclination.

Table 1: Summary of field plot data

Stratum	Selection criterion	Number of field plots	Species composition, percentage pine/spruce/ other	Stem volume, average and 5/95 percentiles (m^3 ha^{-1})	Stem density, average and 5/95 percentiles (ha^{-1})
1	Age 25-74 years, pine dominated (>=60%)	23	61/25/14	40, 28/59	1484, 539/2847
2	Age 25-74 years, spruce dominated	29	0/65/35	49, 13/122	1524, 654/2493
3	Age 25-74 years, mixed forest	33	31/40/29	43, 6/132	1299, 601/2440
4	Age >75 years, spruce dominated	60	9/74/17	119, 41/218	895, 540/1450
5	Age >75 years, pine dominated or mixed forest	34	36/56/8	140, 51/261	895, 413/1577

2.4 Individual tree crown delineation, ITC

The first task was to automatically delineate tree crowns based on geometric tree crown models. A correlation image was produced by using geometric tree models and a Digital Canopy Model (DCM) derived from ALS height data. The correlation image was then smoothed and used for segmentation: a seed was placed at each pixel, with a DCM value greater than the height threshold and with a positive correlation value, and was allowed to climb to the neighbour pixel with the highest correlation value. The pixels with seeds climbing to the same local maximum defined a tree crown segment (Holmgren et al. 2006). The result was crown segments; each included an individual tree or a group of trees. The tree position was estimated by taking the x, y-position of the maximum canopy height value within the segment, and a measure of tree height (H) was achieved from the value of the maximum. The crown area of an individual tree could be derived by counting the number of pixels of a segment. A width (W) of a segment was derived assuming that a tree crown was circular.

2.5 Field plot matching

The three dimensional spatial pattern of the laser detected trees were matched with the spatial pattern of field measured positions of individual trees on a plot. The trees detected in ALS data were automatically linked to field measured trees (Olofsson *et. al* 2008).

2.6 Estimation on tree segment level

Each segment should ideally correspond to one tree on the ground but in reality, one segment may enclose several trees or one tree may be divided into several segments (Figure 1). Single tree properties have been estimated in two different ways: With regression models for variables of one tree in each segment, ITC, and with regression models for variables of the largest tree plus variables of the other trees in the segment, ITC with classification.

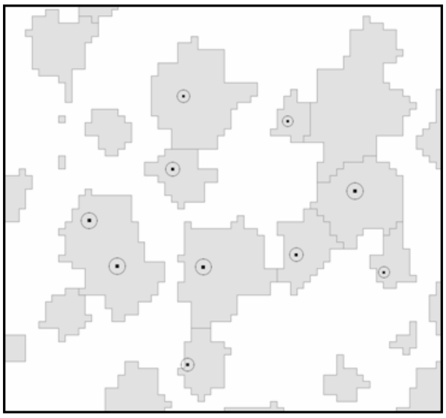

Figure 1: Example of polygons from segmentation of ALS data and field measured trees shown as point symbols and circles with radius proportional to diameter.

2.6.1 Classification of segments to determine number of trees

Features were extracted from ALS data within the segments in order to model the number of field trees within a segment. Only segments where the centre was located inside a field plot and at least 2 m from the boundary were used in the analysis to reduce the number of segments covering ground outside field plots, referred to as reference segments. The variables 1-2 (Table 2) were calculated from laser data, the rest were derived from field data.

Table 2: Variables used for analysis on tree crown segment level

	Variable	Description
1	W	$\sqrt{A}$, A is the area of the segment
2	W/mean(W)	Mean(W) is the mean of segment widths within same plot
3	N	Number of field measured trees within segment
4	D_{max}	Maximum field measured stem diameter found within segment
5	D_{other}	Sum of field measured stem diameter for other trees within segment
6	H_{max}	Maximum field measured tree height found within segment
7	H_{other}	Mean of field measured tree height for other trees within segment
8	B_{max}	Maximum field measured basal area found within segment
9	B_{other}	Sum of field measured basal area for other trees within segment
10	V_{max}	Maximum field measured stem volume found within segment
11	V_{other}	Sum of field measured stem volume for other trees within segment

The strongest correlation for number of field measured trees inside segment was obtained for W and W/mean(W). W and W/mean(W) were divided into eight intervals and each reference segment was placed in an interval in order to estimate the probability for a reference segment to enclose a certain number of trees. The number of segments enclosing 1, 2, 3 and 4 or more field measured trees respectively was calculated for each interval and divided by the total number of segments in the interval.

$$P(N=i)=\frac{n_i}{\sum_{i=1}^{N_{max}} n_i} \tag{1}$$

where n_i = number of segments enclosing i field measured trees in the interval and $N_{max} = 4$. $P(N=i)$ is an estimate of the probability for a segment to enclose i field measured trees. The reference segments were used to build regression functions for 4-11 (Table 2). The regression was done separately for segments enclosing different number of field measured trees.

An unknown segment was first placed in an interval determined by W and W/mean(W). The number of trees inside the segment was estimated as the sum of the probability to have a certain number of trees inside the segment times the number of trees.

$$N_{estimated}=\sum_{i=1}^{N_{max}} P(N=i)\times i \tag{2}$$

2.6.2 Estimation of tree variables from segments

The variables 4-11 (Table 2) were estimated in each segment as

$$A_{estimated}=\sum_{i=1}^{N_{max}} P(N=i)\times A_i \tag{3}$$

where A_i is the value of the variable calculated from a regression model for segments enclosing i field measured trees.

The result for each segment was an estimate of variables for the largest tree in the segment, i.e. 4, 6, 8 and 10 (Table 2). ITC with classification also resulted in an estimate of variables for the rest of the trees in the segment, i.e. 5, 7, 9 and 11 (Table 2). The later estimate was divided by the estimated number of trees minus one to get an estimate for each tree.

$$A'_{estimated} = A_{estimated}/(N_{estimated} - 1) \tag{4}$$

If the result for the tree diameter was below the minimum value for the field measured tree diameter, $N_{estimated}$ was iteratively reduced with one until the resulting tree diameter was above the minimum value. If the tree diameter was too small even when divided by one, the estimate was discarded. The estimates for the largest tree and the rest of the trees were put in a list of tree candidates.

2.7 Estimation on raster cell level

2.7.1. Estimation based on laser canopy height distribution, LCH

Several features, height percentiles, average height of laser reflections, standard deviation of laser reflections and vegetation ratio, were derived based on the Laser Canopy Height (LCH) distribution by using vegetation returns. In order to exclude returns from below the canopy, e.g. shrubs and stones, vegetation returns were defined as returns with a vertical distance to the DTM greater than one meter and 10% of the maximum height within the plot/raster cell. These features were used to build regression models for the field measured percentiles for stem diameter and height distributions at 25%, 50%, 75% and 100%, as well as average stem volume per hectare and number of stems per hectare. Stepwise regression was used to find the most significant variables and Seemingly Unrelated Regression (SUR) was finally used to model the percentiles (Table 3).

Table 3: Seemingly unrelated regression (SUR) for tree height and stem diameter distribution using laser canopy height distribution

SUR model for stem diameter percentiles	SUR model for tree height percentiles
$D_{25} \sim p_{10}+p_{70}+z_{avg}+z_{stdh}+vegratio$	$H_{25} \sim p_{10}+ p_{70}+z_{avg}+z_{stdh}+vegratio$
$D_{50} \sim p_{70}+z_{avg}+z_{stdh}+vegratio$	$H_{50} \sim p_{70}+z_{avg}+z_{stdh}+vegratio$
$D_{75} \sim p_{70}+z_{avg}+z_{stdh}+vegratio$	$H_{75} \sim p_{95}+z_{avg}+z_{stdh}+vegratio$
$D_{100} \sim p_{95}+z_{avg}+z_{stdh}+vegratio$	$H_{100} \sim p_{95}+z_{avg}+z_{stdh}+vegratio$

The regression model used to estimate stem volume per hectare was (Eq. 5) and the regression model used to estimate number of stems per hectare was (Eq. 6). The result was corrected for logarithmic bias (Holm, 1977).

$$\log(Vol) \sim \log(p_{90}) + \log(vegratio) + \log(z_{avg}) \tag{5}$$

$$Dens \sim p_{90} + z_{stdh} + vegratio \tag{6}$$

where D_{25}, D_{50}, D_{75} and D_{100} = 25, 50, 75 and 100 percentile from field measured tree diameter, H_{25}, H_{50}, H_{75} and H_{100} = 25, 50, 75 and 100 percentile from field measured tree height, Vol = stem volume per hectare, Dens = number of stems per hectare, p_{10}, p_{20}, … = 10, 20, … percentile from laser reflection heights on plot, z_{avg} = average height of laser reflections on plot and z_{stdh} = standard deviation of laser reflections on plot and vegratio = vegetation ratio, the number of laser reflections from vegetation divided by the total number of laser reflections on plot.

2.7.2. Estimation based on laser canopy height distribution, LCH, and distribution of detected trees, ITC

The laser reflection variables were combined with variables from individual tree detection aggregated on plots and used to build regression models for the field measured variables. Stepwise regression was used to find the most significant variables and SUR was used to model the percentiles (Table 4).

Table 4: Seemingly unrelated regression (SUR) for tree height and stem diameter distribution using distribution of detected trees

SUR model for stem diameter percentiles	SUR model for tree height percentiles
$D_{25} \sim p_{10}+z_{avg}+vegratio+D_{60}(ST)+H_{50}(ST)$	$H_{25} \sim p_{10}+z_{avg}+vegratio+D_{60}(ST)+H_{50}(ST)$
$D_{50} \sim p_{30}+z_{avg}+vegratio+D_{60}(ST)+H_{75}(ST)$	$H_{50} \sim p_{30}+z_{avg}+vegratio+D_{60}(ST)+H_{75}(ST)$
$D_{75} \sim p_{70}+z_{avg}+vegratio+D_{75}(ST)+H_{80}(ST)$	$H_{75} \sim p_{70}+z_{avg}+vegratio+D_{75}(ST)+H_{80}(ST)$
$D_{100} \sim p_{95}+z_{avg}+vegratio+D_{75}(ST)+H_{100}(ST)$	$H_{100} \sim p_{95}+z_{avg}+vegratio+D_{75}(ST)+H_{100}(ST)$

The regression model used to estimate stem volume per hectare was (Eq.7) and the regression model used to estimate number of stems per hectare was (Eq. 8).

$$Vol \sim vegratio + Vol(ST) \tag{7}$$

$$Dens \sim p_{90} + vegratio + Dens(ST) + H_{100}(ST) \tag{8}$$

where $D_{10}(ST)$, $D_{20}(ST)$, … = 10, 20, … percentile from diameters from individual tree detection, $H_{10}(ST)$, $H_{20}(ST)$, … = 10, 20, … percentile from heights from individual tree detection, Vol(ST) = stem volume per hectare from individual tree detection and Dens(ST) = number of stems per hectare from individual tree detection.

2.8 Adjusting tree candidate list from estimates on raster cell level

The estimates of tree diameter and height percentile, stem volume and stems per hectare were used to identify plots with similar distributions. This was done by finding the plots with the smallest sums of squared differences between the values, i.e. the nearest neighbours. Plots were included in the list one by one until the number of trees was at least 800 or the number of included plots was 10.

The field measured trees on plots with similar distributions were put into a field distribution matrix where each row corresponded to a tree height percentile and each column to a tree diameter percentile (Table 5). The percentiles were calculated from the list of trees on similar plots.

Table 5: The distribution matrixes used for the analysis

Field distribution					Target distribution					Stem distribution				
n_{f11}	n_{f12}	n_{f13}	n_{f14}	n_{f15}	n_{t11}	n_{t12}	n_{t13}	n_{t14}	n_{t15}	n_{st11}	n_{st12}	n_{st13}	n_{st14}	n_{st15}
n_{f21}	n_{f22}	n_{f23}	n_{f24}	n_{f25}	n_{t21}	n_{t22}	n_{t23}	n_{t24}	n_{t25}	n_{st21}	n_{st22}	n_{st23}	n_{st24}	n_{st25}
n_{f31}	n_{f32}	n_{f33}	n_{f34}	n_{f35}	n_{t31}	n_{t32}	n_{t33}	n_{t34}	n_{t35}	n_{st31}	n_{st32}	n_{st33}	n_{st34}	n_{st35}
n_{f41}	n_{f42}	n_{f43}	n_{f44}	n_{f45}	n_{t41}	n_{t42}	n_{t43}	n_{t44}	n_{t45}	n_{st41}	n_{st42}	n_{st43}	n_{st44}	n_{st45}
n_{f51}	n_{f52}	n_{f53}	n_{f54}	n_{f55}	n_{t51}	n_{t52}	n_{t53}	n_{t54}	n_{t55}	n_{st51}	n_{st52}	n_{st53}	n_{st54}	n_{st55}

The number of trees n_{fij} in each interval was multiplied by a scaling factor.

$$S_{vol} = \frac{Volume\ on\ plot\ from\ area\ estimates}{Total\ volume\ on\ similar\ plots} \quad (9)$$

The result was an estimated target distribution matrix where each element corresponded to a tree diameter and height percentile and the value n_{tij} corresponded to the number of trees on the plot in each percentile. The distribution of tree candidates was calculated by summing the number of tree candidates n_{stij} in each percentile given by the target distribution. Tree candidates with a tree diameter or height larger than the 100 percentile were excluded from the list.

The difference between the target distribution and the candidate distribution was calculated for each interval. If the number of tree candidates was too big, that number of tree candidates was excluded from the list. If the number of tree candidates was too small, that number of trees with correct tree diameter and height was added to the list by selecting trees at random from the list of field measured trees. The result was a list of trees with distribution and stem volume on plots predicted by the estimates on plot level.

The result was aggregated on plot level and the procedure was repeated 50 times to study the average accuracy of the estimation.

2.9 Validation

RMSE and bias of stem volume per hectare and stem number per hectare was calculated for each method. Error index for tree heights, diameters and basal area on each plot was also calculated. The error index EI is defined as (Reynolds *et al.* 1988),

$$EI = \frac{1}{N_T}\sum_{j=1}^{m}\left|T_j - I_j\right| \quad (10)$$

where I_j is the number of estimated trees to histogram class j, T_j is the number of actual trees in class j, and N_T is the total number of actual trees. This index measures the proportion of mismatch between two histograms based on given class boundaries.

3. Results

For estimation of stem volume, marginally lower RMSE was obtained from the model based on individual tree crown delineation after accumulation to plot level (Table 6, A) compared to the area based method that used laser canopy height percentiles as explanatory variables in the regression model (Table 6, C). For estimation of number of stems, RMSE was lower for the method based on LCH distribution compared to the ITC based method. ITC resulted in a large negative bias which was reduced to zero by using LCH.

Both RMSE and bias was reduced for the ITC based method by classification of segments (Table 6, A and B). Further reduction of RMSE was possible if the estimates on segment level were summed to plot level and then used together with vegetation ratio as explanatory variables in a linear regression model (Table 4 and Eq. 7). By using this method (Table 6, D) for estimation of stem volume, the bias could be reduced to zero and the lowest RMSE was obtained. Adjusting the tree lists with the diameter-height distribution target matrix did not change the volume estimates much. For the stem number estimates, both RMSE and bias were reduced (Table 6, E and F).

Table 6: RMSE and bias for stem volume and stem number estimates on plot level using the methods: Individual Tree Crown delineation (ITC), ITC with classification, Laser Canopy Height (LCH) distribution, LCH and ITC distributions, ITC with adjustment, and ITC with classification and adjustment. Percentages of mean values within brackets

	Method	Stem volume (m^3ha^{-1})		Stem number (ha^{-1})	
		RMSE	Bias	RMSE	Bias
A	ITC	35 (36%)	-14 (-14%)	595 (52%)	-403 (-35%)
B	ITC with classification	33 (34%)	-2 (-3%)	515 (45%)	-208 (-18%)
C	LCH distribution	35 (37%)	-2 (-2%)	358 (31%)	0 (0%)
D	LCH and ITC distribution	31 (33%)	0 (0%)	339 (30%)	-2 (0%)
E	ITC with adjustment	34 (36%)	4 (4%)	402 (35%)	44 (4%)
F	ITC with classification and adjustment	33 (34%)	4 (4%)	411 (36%)	52 (5%)

The error index, which measures the proportion of mismatch between two histograms, decreased after adjustment of tree lists with the diameter-height distribution target matrix. This was observed for tree height, stem diameter, and basal area distributions, although the different was most obvious for tree height and stem diameter distributions (Table 7).

Table 7: Error index for distribution of tree height, stem diameter, and basal area, on plot level using the methods: Individual Tree Crown delineation (ITC), ITC with classification, ITC with adjustment, and ITC with classification and adjustment.

	Method	Error index		
		Tree height	Stem diameter	Basal area
A	ITC	98	97	90
B	ITC with classification	109	99	92
E	ITC with adjustment	95	92	89
F	ITC with classification and adjustment	96	93	89

4. Discussion

This study examined combinations of area based estimations and single tree estimations from segmentation. Such a combination gives more accurate estimation of stem volume per hectare than area based estimations only. The result for stem volume from LCH area based estimation was slightly less accurate than the result from ITC but it is not possible to draw any conclusions from that since the difference was small. RMSE was higher for all methods compared to other studies (Maltamo et al. 2006, Næsset et al. 2004). One reason may be that the plot size was small. Trees standing close to a plot boundary may have a big part of their branches on the other side. It is likely that the overall accuracy would be higher if a larger plot size was used. The proportion of deciduous forest was high, approximately 30% in stratum 2 and 3, which may degrade the accuracy considerably (Næsset *et al.* 2004). The analyses are not done with stratified data. Stratification of data and use of separate regression models for different strata may improve the accuracy of the estimates. However, the aim of this study is to compare the different methods and their results relative to each other using the same dataset.

The study has also proposed a new method to create tree lists from estimation of forest variables on raster cell level. Those tree lists are more accurate estimates of stems per area unit than tree lists from individual tree detection. However, the RMSE of stem volume per area unit is almost the same. This may be due to the random selection from the list of field measured trees.

The error index was lower for the tree lists adjusted with results from the area based method. Individual tree detection works best for larger trees and the area based method probably adds most information for smaller trees. It may be possible to improve this by deriving larger trees from individual tree detection and adjusting the distribution according to results from the area based method for smaller trees. This is in line with the method used by Maltamo *et al.* (2004).

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